



GCSE MARKING SCHEME

AUTUMN 2024

**GCSE
MATHEMATICS
UNIT 2 – HIGHER TIER
3300U60-1**

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCSE MATHEMATICS
AUTUMN 2024 MARKING SCHEME

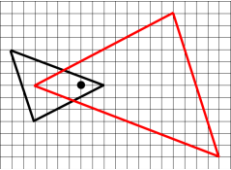
GCSE Mathematics Unit 2: Higher Tier	Mark	Comments
1. (Number of 50p coins =) $(£)19.20 \div (£)0.20 \div 8 \times 5$ or equivalent (Number of 50p coins =) 60 (Value of 50p coins = $60 \times (£)0.5(0)=$ $(£)30(.00)$) (Total value of coins = $(£)19.20 + (£)30 =$) $(£)49.2(0)$)	M2 A1 A1 A1	May be seen in stages. May be seen in any order. Award M1 for one of the following: • $(£)19.20 \div 8 \times 5 (= (£)12)$ • 96 (number of 20p coins $(£)19.20 \div (£)0.20$ or 19.20×5) • $(£)19.20 \div (£)0.20 \div 8 (=12)$ • 'their derived number of 20p coins' $\div 8$ • 12 • $(£)19.20 \div 8$ • $(£)2.4(0)$ CAO. May be implied in further working. FT 'their 60' $\times (£)0.5(0)$ provided M2 awarded and 'their 60' is rounded or truncated if required to a whole number of coins. May be implied in later working. FT provided M2 or M1 awarded $(£)19.2(0) + \text{'their 60'} \times (£)0.5(0)$. If no marks, award SC1 for sight of $(8 \times (£)0.2(0)=) \underline{£1.60}$ and $(5 \times (£)0.5(0)=) \underline{£2.50}$ or equivalent.
Organisation and Communication Accuracy of writing	OC1 W1	For OC1, candidates will be expected to: • present their response in a structured way • explain to the reader what they are doing at each step of their response • lay out their explanation and working in a way that is clear and logical • write a conclusion that draws together their results and explains what their answer means For W1, candidates will be expected to: • show all their working • make few, if any, errors in spelling, punctuation and grammar • use correct mathematical form in their working • use appropriate terminology, units, etc.

2. One correct evaluation $0 \leq x \leq 1$ 2 correct evaluations $0.65 \leq x \leq 0.85$, (one value < 0, one value > 0) 2 correct evaluations $0.65 \leq x \leq 0.75$, (one value < 0, one value > 0) $x = 0.7$	B1 B1 M1 A1	Correct evaluation regarded as enough to identify if < 0 or > 0. If evaluations not seen accept 'too high' or 'too low'. <table><tr><td>x</td><td>$x^3 + 5x - 4$ (check $x^3 + 5x = 4$)</td><td></td><td></td></tr><tr><td>0</td><td>-4</td><td></td><td></td></tr><tr><td>0.1</td><td>-3.499</td><td></td><td></td></tr><tr><td>0.2</td><td>-2.992</td><td></td><td></td></tr><tr><td>0.3</td><td>-2.473</td><td></td><td></td></tr><tr><td>0.4</td><td>-1.936</td><td></td><td></td></tr><tr><td>0.5</td><td>-1.375</td><td></td><td></td></tr><tr><td>0.6</td><td>-0.784</td><td>0.65</td><td>-0.475</td></tr><tr><td>0.7</td><td>-0.157</td><td>0.73</td><td>0.039</td></tr><tr><td>0.8</td><td>0.512</td><td>0.74</td><td>0.105</td></tr><tr><td>0.9</td><td>1.229</td><td>0.75</td><td>0.172</td></tr><tr><td>1</td><td>2</td><td>0.85</td><td>0.864</td></tr></table> Unsupported $x = 0.7$ is awarded B0B0M0A0. An answer of $x = 0.7$ can only be awarded B1B1M1A1, following sight of 2 correct evaluations $0.65 \leq x \leq 0.75$ with one evaluation < 0 and one evaluation > 0.	x	$x^3 + 5x - 4$ (check $x^3 + 5x = 4$)			0	-4			0.1	-3.499			0.2	-2.992			0.3	-2.473			0.4	-1.936			0.5	-1.375			0.6	-0.784	0.65	-0.475	0.7	-0.157	0.73	0.039	0.8	0.512	0.74	0.105	0.9	1.229	0.75	0.172	1	2	0.85	0.864
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3. Five values that satisfy both conditions: - Total = 22.5 - Unique mode = 3.5 <table><tr><td>3.5</td><td>3.5</td><td colspan="3">3 different values that total 15.5</td></tr><tr><td>3.5</td><td>3.5</td><td>3.5</td><td colspan="2">2 values that total 12</td></tr><tr><td>3.5</td><td>3.5</td><td>3.5</td><td>3.5</td><td>8.5</td></tr></table>	3.5	3.5	3 different values that total 15.5			3.5	3.5	3.5	2 values that total 12		3.5	3.5	3.5	3.5	8.5	B3	May be written in any order. Answer boxes take precedence. Award B2 for one of the following: 5 values that total 22.5 - sight of 5×4.5 and 5 values with a unique mode of 3.5 - sight of 22.5 and 5 values with a unique mode of 3.5. Award B1 for one of the following: - sight of 5×4.5 - sight of 22.5 5 values with a unique mode of 3.5. If answer boxes are blank, the 5 values need to be clearly identified in the working space.																																	
3.5	3.5	3 different values that total 15.5																																																
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4. (Number of sides =) $\frac{360}{180 - 171}$ $= 40$ (sides)	M2 A1	Award M1 for sight of one of the following: $180 - 171$ - an appropriate 9. CAO. Allow an embedded answer but penalise -1 if contradicted by number of sides $\neq 40$.																																																
4. Alternative method $\frac{(n - 2) \times 180}{n} = 171$ or $180n - 360 = 171n$ or equivalent $9n = 360$ or equivalent $n = 40$ (sides)	M1 m1 A1	Brackets may be implied by later correct work. CAO. Allow an embedded answer but penalise -1 if contradicted by number of sides $\neq 40$.																																																

5.(a) $(AC^2 =) 13 \cdot 5^2 + 10 \cdot 8^2$ or equivalent $(AC =) \sqrt{13 \cdot 5^2 + 10 \cdot 8^2}$ or equivalent (AC =) 17.3 or 17.2(88...) or 17.29 or $\frac{27\sqrt{41}}{10}$ (cm)	M1 m1 A1	Check diagram for answers. (AC² =) 182.25 + 116.64 = 298.89. (AC =) $\sqrt{298.89}$. Sight of $\sqrt{\text{'their 298.89'}}$ or $\sqrt{\text{'their 298.89'}}$ evaluated is awarded m1 provided M1 previously gained. CAO. Mark final answer. Allow 17 provided from correct workings. Final answer of AC = 298.89 is M1m0A0. Accept a rounded or truncated answer. An unsupported correct answer is awarded M1m1A1.						
<u>5.(a) Alternative method</u> Correct use of a two-step trigonometric method (AC =) 17.3 or 17.2(88...) or 17.29 or $\frac{27\sqrt{41}}{10}$ (cm)	M2 A1	A partial trigonometric method is awarded M0. CAO. Mark final answer. Allow 17 provided from correct workings. Accept a rounded or truncated answer.						
5. (b) $(x =) \tan^{-1} \frac{19.8}{8.7}$ An answer in the range 66.2 to 66.32	M2 A1	Check diagram for answers. Award M1 for $\tan x = \frac{19.8}{8.7} (= 2.275(8..))$ Mark final answer. Allow 66 provided from correct workings. Accept a rounded or truncated answer. Allow correct angles given in radians or gradians. <table border="1" data-bbox="858 1182 1497 1272"> <tr> <th></th><th>Radians</th><th>Gradians</th></tr> <tr> <td>$\tan^{-1} \frac{19.8}{8.7}$</td><td>1.1567...</td><td>73.6440...</td></tr> </table>		Radians	Gradians	$\tan^{-1} \frac{19.8}{8.7}$	1.1567...	73.6440...
	Radians	Gradians						
$\tan^{-1} \frac{19.8}{8.7}$	1.1567...	73.6440...						
<u>5.(b) Alternative method</u> Correct use of a two-step trigonometric method An answer in the range 66.2 to 66.32	M2 A1	A partial trigonometric method is awarded M0. Mark final answer. Allow 66 provided from correct workings. Accept a rounded or truncated answer. Allow correct angles given in radians or gradians.						

6. Correct bisector of 60° Arc of radius 5 cm, centre B. Correct region identified.	B2 B1 B1	Allow a tolerance of $\pm 2^\circ$ and ± 2 mm. The construction need not be below the line for the B2 and the first B1 (but the final B1 won't be awarded). Allow at A or B. Correct construction arcs (initial and secondary) must be seen or an alternative valid method. The angle must be formed for the bisector and the line must reach the intersection of the arcs. Award B1 for one of the following: • correct construction of 60° at A or B. The initial and secondary construction arcs must be seen but line forming the angle may not need to be seen (depending on method) • correct bisector of 'their acute angle' at A or B. The initial and secondary construction arcs and bisector line must be seen and reach the intersection of the arcs. For B1, the arc must be of sufficient length so as not to be considered a 'point' or a 'notch' i.e. for a sector of at least 10° at B. For this B1, the region must be: • in the correct location at B • below the line • include an angle • include an arc. FT if at least B1 previously awarded.
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7.(a) Sight of 0.032 or equivalent e.g. 3.2% or $\frac{160}{5000}$ Explanation e.g. 'all data used', 'last point plotted', 'the number of batteries checked was the highest'.	B1 E1	Answer line takes precedence. ISW. Allow incorrect probability notation e.g. 160 out of 5000. Accept any indication that the final reading should give the best estimate. Allow 'the more times you repeat an action, the more accurate the estimate is'.										
7.(b) $3000 \times 0.033 \times 0.026$ or $3000 \times 0.033 \times 2.6$ (£)2.57(4) or 257(.4)(p) ISW	M2 A1	May be done in any order. May be seen in stages or implied in later working. Award M1 for sight of one of the following, allowing one place value error in 0.033 or 0.026 (may be embedded): • 3000×0.033 • 99 • $3000 \times (\text{£})0.026$ • $3000 \times 2.6(\text{p})$ • (£)78 or 7800(p) • 0.026×0.033 • 2.6×0.033 • 0.000858 • 0.0858. Award M1 for sight of the consecutive digits 99 or 78 or 858 in a number involving a place value error e.g. 990 but not 8990. CAO. Allow (£)2.58 or 258(p) (If units are given they must be correct). Incorrect units may imply M2 A0. Award M1 A0 for one of the following answers: <table><tr><th>Answer</th><th>From</th></tr><tr><td>2(.)34</td><td>$(3000 \times 0.026 \times 0.03)$</td></tr><tr><td>2(.)10(6) or 2(.)11</td><td>$(3000 \times 0.026 \times 0.027)$</td></tr><tr><td>2(.)62(2)</td><td>$(3000 \times 0.026 \times 0.029)$</td></tr><tr><td>2(.)49(6) or 2(.)50</td><td>$(3000 \times 0.026 \times 0.032)$</td></tr></table>	Answer	From	2(.)34	$(3000 \times 0.026 \times 0.03)$	2(.)10(6) or 2(.)11	$(3000 \times 0.026 \times 0.027)$	2(.)62(2)	$(3000 \times 0.026 \times 0.029)$	2(.)49(6) or 2(.)50	$(3000 \times 0.026 \times 0.032)$
Answer	From											
2(.)34	$(3000 \times 0.026 \times 0.03)$											
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2(.)62(2)	$(3000 \times 0.026 \times 0.029)$											
2(.)49(6) or 2(.)50	$(3000 \times 0.026 \times 0.032)$											

8. (Area of top or bottom =) $\pi \times 10^2 (\times 2)$ (Curved surface area =) $2 \times \pi \times 10 \times \text{height}$ or equivalent (e.g. $62.8(31...) \times h$) (Height =) $\frac{1570.8 - \pi \times 10^2 \times 2}{2 \times \pi \times 10}$ (= $\frac{942.48}{62.83}$) or $\frac{1570.8}{2 \times \pi \times 10} - 10$ or equivalent Answer between 14.99 and 15.02 (cm) inclusive	M1 M1 M2 A1	May be seen or implied in later working. Award M1 for sight of an appropriate answer between either: • 314 and $314.2 \text{ (cm}^2\text{)}$ inclusive • 628 and $628.4 \text{ (cm}^2\text{)}$ inclusive. May be seen or implied in later working. Award M2 for isolating the height term. FT for M2 or M1 for 'their $\pi \times 10^2 \times 2$ and 'their $\pi \times 20$', provided that: • π used when calculating the areas • 'their $\pi \times 10^2 \times 2 < 1570.8$ (including using radius = 5cm and diameter = 10cm). Award M1 for equating the three areas to 1570.8 but not isolating the height term e.g. • $\pi \times 10^2 \times 2 + \pi \times 20 \times \text{height} = 1570.8$ • $200\pi + 20\pi \times \text{height} = 1570.8$ • $628.32 + 62.83 \times \text{height} = 1570.8$ • $62.83 \times \text{height} = 942.48$ • $20\pi \times \text{height} = 942.48$ • $2\pi \times 10(10 + \text{height}) = 1570.8$ CAO.
9. (a) $45x + 23y = 89520$ or $23y + 45x = 89520$	B1	May be seen in part (b) as long as not contradicted by an incorrect equation in part (a). Award B1 if 89520 or $45x + 23y$ seen in the table in (a), but $45x + 23y = 89520$ seen in (b).
9.(b) Method to eliminate one variable e.g. equal coefficients <u>AND an appropriate intention to subtract or add (whichever is appropriate)</u> or use a method of substitution. First variable found (The number of seated tickets sold, $x =$) 1560 or (The number of standing tickets sold, $y =$) 840 Second variable found.	M1 A1 A1	No marks for 'trial and improvement'. No marks for unsupported answers. Answer lines take precedence. FT 'their equation' from (a) if of equivalent difficulty (e.g. both the coefficients of x and y are $\neq 0$ and $\neq 1$). Allow one error in one term (not the term with equal coefficients). CAO FT substitution of their '1st variable' evaluated correctly, provided M1 gained. If both correct answers are seen in working space, but contradicted on answer lines, award M1A1A0. Treat reversed answers as a slip (M1A1A1).
10. 	B2	B1 for any one of the following: • 2 correct vertices within a triangle • 3 correct vertices in the correct location not joined to form the triangle • triangle of correct shape, size and orientation in incorrect position • consistent correct use of an incorrect negative scale factor.

14. $((2x + 3) + 4)((2x + 3) - 4)$ $(2x + 7)(2x - 1)$	B2 B2	B1 for $((2x + 3) \dots 4)((2x + 3) \dots 4)$ Mark final answer. Accept $(-2x - 7)(1 - 2x)$ OR $-(2x + 7)(1 - 2x)$. FT provided B1 previously awarded. Ignore an attempt to solve their quadratic set to '=0'.
<u>14. Alternative method</u> $4x^2 + 12x - 7$ $(2x + 7)(2x - 1)$	B2 B2	B1 for sight of $4x^2 + 6x + 6x + 9$ (-16) or equivalent. Mark final answer. Accept $(-2x - 7)(1 - 2x)$ OR $-(2x + 7)(1 - 2x)$. FT for B2 or B1 provided B1 previously awarded and -16 used. Award B1 for one of the following: $(2x \dots 7)(2x \dots 1)$ - two brackets that multiply to give $4x^2 + 12x + k$, $k \neq 0$ - two brackets that multiply to give $4x^2 + mx - 7$, $m \neq 0$, e.g. $(4x \pm 7)(x \mp 1)$ OR $(4x \pm 1)(x \mp 7)$. Ignore an attempt to solve 'their quadratic' set to '=0'.
15. $11x^2 - 23x - 19 = 0$ or $0 = -11x^2 + 23x + 19$ $x = \frac{-(-23) \pm \sqrt{(-23)^2 - 4 \times 11 \times -19}}{2 \times 11}$ OR $x = \frac{-23 \pm \sqrt{(23)^2 - 4 \times -11 \times 19}}{2 \times -11}$ $x = \frac{23 \pm \sqrt{1365}}{22}$ $x = 2.72$ with $x = -0.63$ (answers to 2dp)	B1 M1 A1 A1	'= 0' required unless $a = 11, b = -23, c = -19$ OR $a = -11, b = 23, c = 19$ is used in the quadratic formula. FT only if the rearrangement set to '=0' is seen unless $a = 11, b = -23, c = -19$ OR $a = -11, b = 23, c = 19$ is used in the quadratic formula. This substitution into the formula must be seen for M1, otherwise award M0A0A0. FT 'their derived quadratic equation equated to zero' provided of equivalent difficulty (a, b and c must be non-zero). Allow one slip in substitution for M1 only , but must be correct formula. This can be awarded as a single attempt which may be seen anywhere in the solution for solving their quadratic equation equated to zero. Can be implied from at least one correct value of x evaluated (not necessarily rounded to 2dp), provided M1 awarded. CAO for their quadratic equation.
16. (nth term $\Rightarrow$) $3n^2 - 1$ or equivalent (250 th term $\Rightarrow$) 187499	B2 B1	B1 for sight of $3n^2$ OR $6n^2/2$. Sight of 2 nd difference 6 is insufficient for B1. FT from their $3n^2 \pm k$, where $k \neq 0$ OR from their $3n^2 \pm an$, where $a \neq 0$ OR from their $3n^2 \pm an \pm k$, where $a \neq 0, k \neq 0$.

17.	m^{96}	B1	
18.	(1 – P(all 3 colours are chosen) =) $1 - 6 \times \frac{3}{15} \times \frac{5}{14} \times \frac{7}{13}$ $= \frac{10}{13} \left(= \frac{2100}{2730} \right)$	M3 M2 for $6 \times \frac{3}{15} \times \frac{5}{14} \times \frac{7}{13} \left(= \frac{3}{13} = \frac{630}{2730} \right)$ M1 for $1 - \frac{3}{15} \times \frac{5}{14} \times \frac{7}{13} \left(= \frac{25}{26} = \frac{2625}{2730} \right)$ A1 CAO ISW Accept a decimal answer of 0.76(9...) or 0.77. If no marks award SC1 for one of the following: - Working with replacement leading to an answer of 61/75 (2745/3375) OR 0.81(3) [may be unsupported]. - Recognising all 21 permutations (and no other) OR recognising up to 27 possible permutations, and then identifying the 21 relevant permutations.	
<u>Alternative method 1</u> (P(2 Red & 1 Not Red) OR P(2 Blue & 1 Not Blue) OR P(2 Yellow & 1 Not Yellow) =) $3 \times \frac{3}{15} \times \frac{2}{14} \times \frac{12}{13} +$ $3 \times \frac{5}{15} \times \frac{4}{14} \times \frac{10}{13} +$ $3 \times \frac{7}{15} \times \frac{6}{14} \times \frac{8}{13} \left(= \frac{304}{455} = \frac{1824}{2730} \right)$ (P(3 Red) OR P(3 Blue) OR P(3 Yellow) =) $\left(\frac{3}{15} \times \frac{2}{14} \times \frac{1}{13} \right) + \left(\frac{5}{15} \times \frac{4}{14} \times \frac{3}{13} \right) + \left(\frac{7}{15} \times \frac{6}{14} \times \frac{5}{13} \right)$ $\left(= \frac{46}{455} = \frac{276}{2730} \right)$ (P(at least 2 counters the same colour) =) $\frac{10}{13} \left(= \frac{2100}{2730} \right)$		M2 May be seen in stages. M1 for: $\left(\frac{3}{15} \times \frac{2}{14} \times \frac{12}{13} \right) + \left(\frac{5}{15} \times \frac{4}{14} \times \frac{10}{13} \right) + \left(\frac{7}{15} \times \frac{6}{14} \times \frac{8}{13} \right)$ $\left(= \frac{12}{455} + \frac{20}{273} + \frac{8}{65} = \frac{304}{1365} \right)$ OR $3 \times \frac{3}{15} \times \frac{2}{14} \times \frac{12}{13} \left(= \frac{36}{455} = \frac{216}{2730} \right) \text{ OR}$ $3 \times \frac{5}{15} \times \frac{4}{14} \times \frac{10}{13} \left(= \frac{20}{91} = \frac{600}{2730} \right) \text{ OR}$ $3 \times \frac{7}{15} \times \frac{6}{14} \times \frac{8}{13} \left(= \frac{24}{65} = \frac{1008}{2730} \right)$ M1 Award M1M1 for an answer of $\frac{34}{105}$ from sight of: P(R,R,other)+ P(B,B,other)+ P(Y,Y,other) OR P(R,R)+ P(B,B)+ P(Y,Y) A1 CAO ISW Accept a decimal answer of 0.76(9...) or 0.77. If no marks award SC1 for one of the following: - Working with replacement leading to an answer of 61/75 (2745/3375) OR 0.81(3) [may be unsupported]. - Recognising all 21 permutations (and no other) OR recognising up to 27 possible permutations, and then identifying the 21 relevant permutations.	

<u>Alternative method 2</u> (P(2 Red & 1 Blue) OR (P(2 Red & 1 Yellow) OR P(2 Blue & 1 Red) OR P(2 Blue & 1 Yellow) OR P(2 Yellow & 1 Red) OR P(2 Yellow & 1 Blue) =) $\left(3 \times \frac{3}{15} \times \frac{2}{14} \times \frac{5}{13}\right) + \left(3 \times \frac{3}{15} \times \frac{2}{14} \times \frac{7}{13}\right) +$ $\left(3 \times \frac{5}{15} \times \frac{4}{14} \times \frac{3}{13}\right) + \left(3 \times \frac{5}{15} \times \frac{4}{14} \times \frac{7}{13}\right) +$ $\left(3 \times \frac{7}{15} \times \frac{6}{14} \times \frac{3}{13}\right) + \left(3 \times \frac{7}{15} \times \frac{6}{14} \times \frac{5}{13}\right)$ $\left(= \frac{304}{455} = \frac{1824}{2730}\right)$ (P(3 Red) OR P(3 Blue) OR P(3 Yellow) =) $\left(\frac{3}{15} \times \frac{2}{14} \times \frac{1}{13}\right) + \left(\frac{5}{15} \times \frac{4}{14} \times \frac{3}{13}\right) + \left(\frac{7}{15} \times \frac{6}{14} \times \frac{5}{13}\right)$ $\left(= \frac{46}{455} = \frac{276}{2730}\right)$ (P(at least 2 counters the same colour) =) $\frac{10}{13} \left(= \frac{2100}{2730}\right)$	M2 M1 A1	May be seen in stages. M1 for: $\left(\frac{3}{15} \times \frac{2}{14} \times \frac{5}{13}\right) + \left(\frac{3}{15} \times \frac{2}{14} \times \frac{7}{13}\right) +$ $\left(\frac{5}{15} \times \frac{4}{14} \times \frac{3}{13}\right) + \left(\frac{5}{15} \times \frac{4}{14} \times \frac{7}{13}\right) +$ $\left(\frac{7}{15} \times \frac{6}{14} \times \frac{3}{13}\right) + \left(\frac{7}{15} \times \frac{6}{14} \times \frac{5}{13}\right)$ $\left(= \frac{304}{1365} = \frac{608}{2730}\right)$ OR $\left(3 \times \frac{3}{15} \times \frac{2}{14} \times \frac{5}{13}\right) + \left(3 \times \frac{3}{15} \times \frac{2}{14} \times \frac{7}{13}\right) \left(= \frac{36}{455} = \frac{216}{2730}\right)$ OR $\left(3 \times \frac{5}{15} \times \frac{4}{14} \times \frac{3}{13}\right) + \left(3 \times \frac{5}{15} \times \frac{4}{14} \times \frac{7}{13}\right) \left(= \frac{20}{91} = \frac{600}{2730}\right)$ OR $\left(3 \times \frac{7}{15} \times \frac{6}{14} \times \frac{3}{13}\right) + \left(3 \times \frac{7}{15} \times \frac{6}{14} \times \frac{5}{13}\right) \left(= \frac{24}{65} = \frac{1008}{2730}\right)$ CAO ISW Accept a decimal answer of 0.76(9...) or 0.77. If no marks award SC1 for one of the following: - Working with replacement leading to an answer of 61/75 (2745/3375) OR 0.81(3) [may be unsupported]. - Recognising all 21 permutations (and no other) OR recognising up to 27 possible permutations, and then identifying the 21 relevant permutations.
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<u>Alternative method 3</u> (P(at least 2 Red) = P(2 Red & 1 Not Red) + P(3 Red)=) $3 \times \frac{3}{15} \times \frac{2}{14} \times \frac{12}{13} + \frac{3}{15} \times \frac{2}{14} \times \frac{1}{13} \quad \left(\frac{37}{455} = \frac{222}{2730} \right)$ (P(at least 2 Blue) = P(2 Blue & 1 Not Blue) + P(3 Blue)=) $3 \times \frac{5}{15} \times \frac{4}{14} \times \frac{10}{13} + \frac{5}{15} \times \frac{4}{14} \times \frac{3}{13} \quad \left(\frac{22}{91} = \frac{660}{2730} \right)$ (P(at least 2 Yellow) = P(2 Yellow & 1 Not Yellow) + P(3 Yellow)=) $3 \times \frac{7}{15} \times \frac{6}{14} \times \frac{8}{13} + \frac{7}{15} \times \frac{6}{14} \times \frac{5}{13} \quad \left(\frac{203}{455} = \frac{1218}{2730} \right)$ (P(at least 2 counters the same colour) =) $\frac{10}{13} \left(\frac{2100}{2730} \right)$	M1 M1 M1 A1	Calculations may be seen in stages e.g. P(2 Red & 1 Not Red) = P(2 Red & 1 Blue) + P(2 Red & 1 Yellow) CAO ISW Accept a decimal answer of 0.76(9...) or 0.77. If no marks award SC1 for one of the following: - Working with replacement leading to an answer of 61/75 (2745/3375) OR 0.81(3) [may be unsupported]. - Recognising all 21 permutations (and no other) OR recognising up to 27 possible permutations, and then identifying the 21 relevant permutations.
19. $\frac{3d}{d+g} = h^2 \quad \text{OR} \quad \sqrt{3d} = h \times \sqrt{d+g}$ $3d = h^2(d+g)$ $3d = h^2d + h^2g \quad \text{OR} \quad -3d = -h^2d - h^2g$ OR $3d - h^2d = h^2g \quad \text{OR} \quad h^2d - 3d = -h^2g$ $d(3 - h^2) = h^2g \quad \text{OR} \quad d(h^2 - 3) = -h^2g$ $d = \frac{h^2g}{(3-h^2)} \quad \text{OR} \quad d = \frac{-h^2g}{(h^2-3)}$	B1 B1 B1 B1 B1	FT until 2nd error for equivalent level of difficulty. Squaring or clearing the fraction. Clearing the fraction from 'their squaring' OR squaring from 'their clearing the fraction'. Expanding the bracket. Factorising terms in d. Isolating d.

20.(a) $(\cos x =) (5^2 + 11^2 - 7^2) / (2 \times 5 \times 11)$ $(= 97/110 \text{ OR } 0.8818...)$ $(x =) 28.1(...^\circ)$	M2 A1	An incorrect trigonometric ratio explicitly stated in the solution is M0, e.g., $\sin x = (5^2 + 11^2 - 7^2) / (2 \times 5 \times 11)$ M1 for $7^2 = 5^2 + 11^2 - 2 \times 5 \times 11 \times \cos x$ Allow an answer of 28° from correct working. <table border="1" data-bbox="858 320 1201 387"> <tr> <th></th><th>Radians</th><th>Gradians</th></tr> <tr> <td>x</td><td>0.49...</td><td>31.26...</td></tr> </table> If no marks, award either: - SC2 for an answer of $78.4(6...^\circ)$ OR 78.5° from working using $CG = 6\text{cm}$ - SC1 for sight of $(\cos x =) (5^2 + 6^2 - 7^2) / (2 \times 5 \times 6)$		Radians	Gradians	x	0.49...	31.26...						
	Radians	Gradians												
x	0.49...	31.26...												
<u>Alternative method by first calculating $\hat{GCH}$</u> A correct and complete method first using the cosine rule to calculate $\hat{GCH}$ and then using the sine rule to calculate x. $(\cos \hat{GCH} =) (7^2 + 11^2 - 5^2) / (2 \times 7 \times 11)$ $(= 145/154 \text{ OR } 0.9415...)$ $(\hat{GCH} =) 19.6(...^\circ)$ $(\sin x =) 7 \times \sin(\hat{GCH}) / 5$ $(= 0.4715...)$ $(x =) 28.1(...^\circ)$	M2 A1	A partial method is M0. <table border="1" data-bbox="858 763 1337 831"> <tr> <th></th><th>Radians</th><th>Gradians</th></tr> <tr> <td>$\hat{GCH}$</td><td>0.343...</td><td>21.872...</td></tr> </table> <table border="1" data-bbox="858 887 1318 954"> <tr> <th></th><th>Radians</th><th>Gradians</th></tr> <tr> <td>x</td><td>0.49...</td><td>31.26...</td></tr> </table> If no marks, award - SC2 for an answer of $78.4(6...^\circ)$ OR 78.5° from working using $CG = 6\text{cm}$ - SC1 for a complete alternative method using length $CG = 6\text{cm}$		Radians	Gradians	$\hat{GCH}$	0.343...	21.872...		Radians	Gradians	x	0.49...	31.26...
	Radians	Gradians												
$\hat{GCH}$	0.343...	21.872...												
	Radians	Gradians												
x	0.49...	31.26...												
<u>Alternative method by first calculating $\hat{CHG}$</u> A correct and complete method first using the cosine rule to calculate $\hat{CHG}$ and then using the sine rule to calculate x. $(\cos \hat{CHG} =) (5^2 + 7^2 - 11^2) / (2 \times 5 \times 7)$ $(= -47/70 \text{ OR } -0.67(14...))$ $(\hat{CHG} =) 132.1(...^\circ)$ $(\sin x =) 7 \times \sin(\hat{CHG}) / 11$ $(= 0.4715...)$ $(x =) 28.1(...^\circ)$	M2 A1	A partial method is M0. <table border="1" data-bbox="858 1357 1337 1424"> <tr> <th></th><th>Radians</th><th>Gradians</th></tr> <tr> <td>$\hat{CHG}$</td><td>2.306...</td><td>146.863...</td></tr> </table> <table border="1" data-bbox="858 1514 1318 1581"> <tr> <th></th><th>Radians</th><th>Gradians</th></tr> <tr> <td>x</td><td>0.49...</td><td>31.26...</td></tr> </table> If no marks, award - SC2 for an answer of $78.4(6...^\circ)$ OR 78.5° from working using $CG = 6\text{cm}$ - SC1 for a complete alternative method using length $CG = 6\text{cm}$		Radians	Gradians	$\hat{CHG}$	2.306...	146.863...		Radians	Gradians	x	0.49...	31.26...
	Radians	Gradians												
$\hat{CHG}$	2.306...	146.863...												
	Radians	Gradians												
x	0.49...	31.26...												

20.(b) (Area of triangle =) $\frac{1}{2} \times 5 \times 11 \times \sin[28.1(\dots)]$ $= 12.9(\dots \text{cm}^2)$ or $13(\text{cm}^2)$ (Major sector area =) $(360 - 28.1(\dots)) / 360 \times \pi \times 5^2$ OR (Area of circle AND Minor sector area =) $\pi \times 5^2$ AND $28.1/360 \times \pi \times 5^2$ $72.36(\text{cm}^2)$ to $72.42(\text{cm}^2)$ OR $78.5(\text{cm}^2)$ to $78.6(\text{cm}^2)$ AND $6.1(\dots \text{cm}^2)$ [Total area = $12.9(\dots) + 72.4(\dots) =$] OR [Total area = $12.9(\dots) + 78.5(\dots) - 6.1(\dots) =$] Answer in the range 85.26 to $85.5(\text{cm}^2)$	M1 A1 M1 A1 B1	FT 'their derived x'. If $CG=6\text{cm}$ is consistently used in part (a), allow a FT with length $CG=6\text{cm}$. FT 'their stated or derived x'. FT 'their derived $12.9(\dots \text{cm}^2)$' and 'their derived $72.4(\dots \text{cm}^2)$' OR 'their derived $12.9(\dots \text{cm}^2)$' and 'their derived $78.5(\dots \text{cm}^2)$' AND 'their derived $6.1(\dots \text{cm}^2)$', provided previous M1M1 awarded.
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